APPENDIX D:
OPTIMIZATION MODEL

The objective of the economic optimization routine is to identify the combination of nutrient control projects included in the model that achieves targeted load reductions to the Chesapeake Bay at the lowest total cost. In this regard, the framework is similar to the other cost-minimization studies, such as Schwartz (2010), which solves for the least-cost nutrient strategy in the Potomac River Basin.

To identify and compare the least-cost solution under alternative scenarios, we formulated the optimization as a mixed integer linear programming (MILP) problem. We solved for the optimal solutions using the General Algebraic Modeling System (GAMS) modeling system. The optimization model uses a branch-and-cut algorithm to search across the possible combinations of the cost and load reduction possibilities and identify the combined set of projects that achieves the targeted aggregate load reduction for the lowest aggregate cost.

The model includes a number of set indices and optional layers of spatial disaggregation:

a = spatial unit defining agricultural areas,

b = BMPs for agricultural and urban abatement options,

i, I = region defining nutrient reduction requirements (e.g., in-state or in-basin)

j = point source facilities,

k = point source abatement technologies,

n = nutrient,

s, S = bay segment,

u = urban spatial unit.

The objective function of the model (depicted by Equation D.1) minimizes the total costs of achieving reduction targets at user-defined regional basis (watershed-wide, in-state, in-basin, or in-basin-state). The sum over all reductions from point, agricultural, and urban sources must meet the established reduction requirement for each region i (Equation D.2).

$$\min \sum_i \begin{pmatrix} \left(\sum_{j \in I} \sum_k \alpha_{jk} * PS\_Costs_{jk} \right) + (1 + Trans\_Cost) * Total\_Ag\_Costs_i \\ + Total\_Urban\_Costs_i \end{pmatrix} \tag{D.1}$$

$$\begin{pmatrix} \left(\sum_{j \in I} \sum_k \alpha_{jk} * PS\_Reductions_{jkn} \right) \\ + Ag\_Reductions_{in} + Urban\_Reductions_{in} \end{pmatrix} \geq Nutrient\_Reduction\_Targets_{in} \tag{D.2}$$

where:

$$Ag\_Reductions_{in} = \sum_{a \in I} \sum_b \left(Area\_Ag_{ab} * Nutrient\_Reduction\_Ag_{ab} \right) / Trade\_Ratio \tag{D.3}$$

$$Urban\_Reductions_{in} = \sum_{u \in I} \sum_b Area\_Urban_{ub} * Nutrient\_Reduction\_Urban_{ub} \tag{D.4}$$

$$\alpha_{jk} = \begin{cases} 1 & \text{if project is adopted.} \\ 0 & \text{otherwise.} \end{cases} \tag{D.5}$$

The terms $Area\_Ag_{ab}$ and $Area\_Urban_{ub}$ are choice variables representing the total acres adopted of BMP *b* agriculture and urban spatial points *a* and *u*, respectively, multiplied by the nutrient reduction per-unit area

Other Constraints

Additional constraints are implemented to ensure a realistic mix of abatement choices. First, a binary choice maximum constraint is adopted for each point source facility that ensures a maximum of one infrastructure project will be adopted at each point source facility.

$$\sum_k \alpha_{jk} \leq 1 \;\; \forall\, j, k \tag{D.6}$$

The next set of constraints restricts total agricultural BMP adoption for each agricultural land segment. Equation D.7 sets an upper bound for each BMP equal to the total land available at each spatial unit, and Equation D.8 ensures that total land conversion out of production is limited to 25% of the current land base. Urban BMP adoption is restricted in a similar fashion (Equations D.9 and D.10), with total area restrictions for the sum of all BMPs (Equation D.8).

$$\sum_b Area\_Ag_{ab} \leq Area\_Max_a \qquad \forall\; a \tag{D.7}$$

$$\sum_{b} Area\_Ag\_Convert_{ab} \leq 0.25 * Area\_Max_{a} \qquad \forall\ a \tag{D.8}$$

$$\sum_{b} Area\_Urban_{ub} \leq Urban\_Area\_Max_{u} \qquad \forall \text{ u} \tag{D.9}$$

$$Area\_Urban_{ub} \leq Urban\_Area\_Max_{u} \qquad \forall \text{ u, b} \tag{D.10}$$

$$Area\_Urban_{u,b \in Restricted} \leq Restricted\_Area_{u,b \in Restricted} \tag{D.11}$$

To ensure local water quality protection, we imposed a restriction on trading behavior at the Bay Segment level (Equation D.12). This is done by ensuring the net difference between achieved reductions within a bay segment and segment-specific reduction requirements from the TMDL is greater than imposed nutrient-specific exceedance requirements. This constraint limits credit buying at the local level and ensures that a portion of required local reductions come through physical abatement choices.

$$\left(\begin{array}{l} \left(\sum_{j \in S} \sum_{k} \alpha_{jk} * PS\_Reductions_{jkn} \right) \\ + Ag\_Reductions\_Segment_{sn} \\ + Urban\_Reductions\_Segment_{sn} \\ \end{array} \right) \geq Segment\_Reduction\_Targets_{sn} - Exceed\_Max_{n} \tag{D.12}$$

Reference

Schwartz, S.S. 2010. "Optimization and Decision Heuristics for Chesapeake Bay Nutrient Reduction Strategies." *Environmental Modeling and Assessment* 15(5):345-359.